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Dynamic Analysis of Marine Cable

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Abstract— This paper will focus on a dynamics analysis of a marine cables which are mainly used in offshore and subsea applications (i.e. Mooring Line, Riser, ROV umbillicals, pipeline etc). We considered an Euler Bernoulli beam cable model which is derived based on Newton's second law of motions and beam cable equilibrium conditions in continuous system. Different beam cable model and end conditions are adapted to recognize our physical problems and also boundary conditions, assumptions are adjusted to obtain solutions. The models are solved both mathematically and numerically. We estimated structural reliable parameters like displacement, natural frequency, damping etc and it is visualized graphically.

Index Terms— Continious System, Euler Bernoulli beam cable model, Mathieu Equation, Cable frequenciy and response, Parametric model.

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1. Introduction

THE dynamic model of the offshore and subsea cable structure discussed have involved only finite number of independent coordinates and ordinary differential equations of motion. In the single degree of freedom systems, one coordinate is chosen to describe the dominant structural vibration mode in the plane. In the multi-degree of freedom systems, examples included the rigid gravity platform with coordinates to describe sliding and rocking motion and jacket template platforms with coordinates to describe the motion of discrete masses lumped at node points.

For line components such as rather long beam, mooring, riser, pipelines, umbillicals and cables, alternative continuous system models may provide more precise and sometimes more economical descriptions of component motion [1]. Since partial differential equation is used to characterize the motion of a continuous line component, solutions are generally more involved mathematically than for a corresponding lumped system. However, if the continuous models are chosen judiciously, closed form expressions can be derived for the characteristic frequenscies and mode shapes of line components, which then lead to upper and lower bounds on their dynamic responses.

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One special class of continuous line components are analyzed in this paper. The structural component considered here is the cable which resists tension but whose bending stiffness is negligible.

2. Governing Equations

The general form the linear Bernoulli-Euler dynamic beam cable equation is derived [1] [2].

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 v}{\partial x^2}\right)-\frac{\partial}{\partial x}\left(P\frac{\partial v}{\partial x}\right)+\bar{c}\frac{\partial v}{\partial t}+\bar{m}\frac{\partial^2 v}{\partial t^2}=\bar{q}(x,t)$$

---------------------- (1)

Where *EI*, *P*, and $\bar{m}$ are arbitrary functions of *x* and excitation load $\bar{q}(x,t)$ is arbitrary in both *x* and *t*.

Cable Frequencies and Mode shape:

For undamped, flexible cable (*EI* =0), a subjected to tension load *P* which is independently of *x* and equation (1) becomes,

$$-P\frac{\partial^2 v}{\partial x^2}+\bar{m}\frac{\partial^2 v}{\partial t^2}=\bar{q}(x,t)$$ ----------------- (2)

Consider the free, undamped vibrations of a flexible cable of constant m and constant tension P= P. The corresponding equation of motion is deduced as,

$$P_0\frac{\partial^2 v}{\partial x^2}-\bar{m}\frac{\partial^2 v}{\partial t^2}=0$$ ----------------------- (3)

The cable frequencies are given by,

$$\omega_n=\frac{n^2\pi^2}{\ell^2}\sqrt{\frac{EI}{\bar{m}}},\quad n=1,2,\ldots$$ ---------------- (4)

And corresponding mode shape is given by,

$$X_n = C_n \sin\frac{n\pi x}{\ell}, \quad n = 1, 2, \ldots \quad \text{-------- (5)}$$

Cable Response (Parametric excitation):
Cable response due to longitudinal or parametric end excitation may occur vertical lines or chain when the ship or buoy undergoes heave motion in regular wave.

$$-(P_0 + P_1 \cos\bar{\omega}t)\frac{\partial^2 v}{\partial x^2} + \bar{m}\frac{\partial^2 v}{\partial t^2} = 0 \quad \text{---------------- (6)}$$

The analytical result of the equation (6) can be expressed interm of Mathieu equation which is [4],

$$\frac{d^2 y_n(\tau)}{d\tau^2} + (\bar{\alpha}_n + \bar{\beta}_n \cos\tau)\, y_n(\tau) = 0 \quad \text{----------------------- (7)}$$

3. WORKED EXAMPLE

This worked example illustrates the mode shape and frequencies of the undamped, free vibration frequencies for a submerged floating marine structure where Riser is connected with BOP or Mooring line is fixed at seabed in fig. a, offshore jacket, jack up, articulated tower platform end of brace is welded to a leg, the ends are not simple supports, but because of leg and joints flexibility, these end are not fully clamped either fig. c, floating pipe line for offloading oil gas to supply vessel or floating water supply

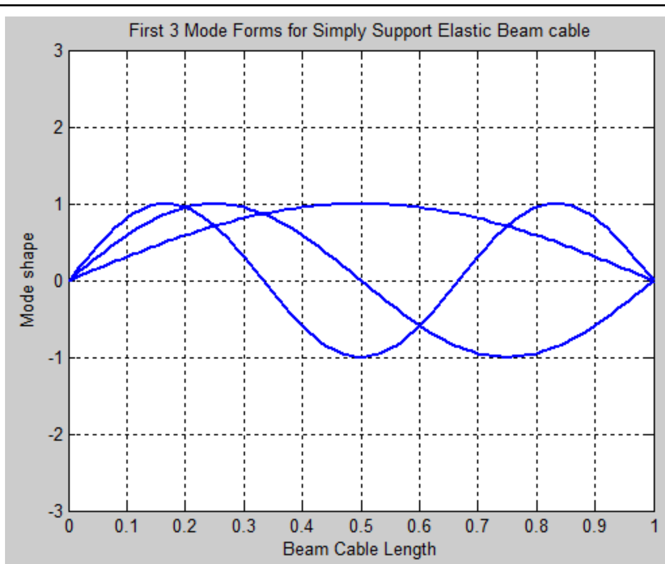

Fig. 1 The first three mode shape for a simply supported beam cable

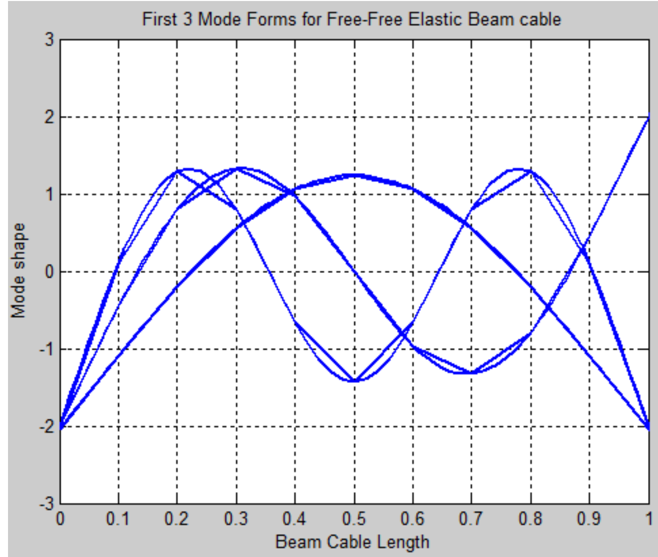

Fig. 2 The first three mode shape for free- free elastic beam cable

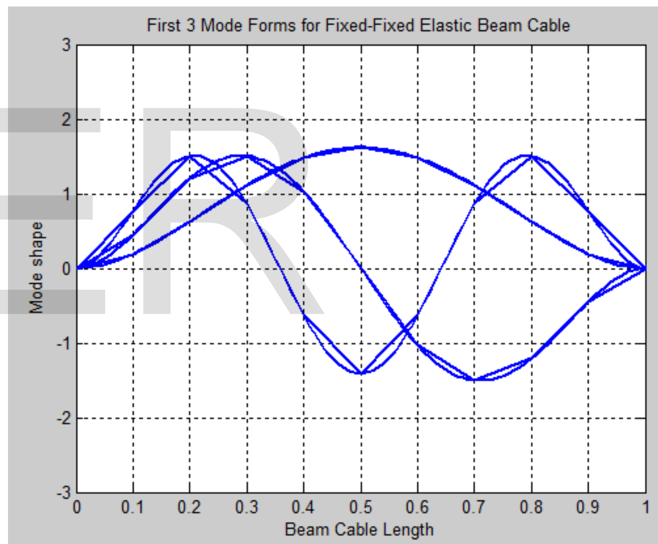

Fig. 3 The first three mode shape for fixed fixed elastic beam cable

pipeline in fig. b, ROV cable fixed with manned structure and other end connected with unmanned vehicle in fig.c which may be considered as free end or pipeline segment during deployment in fig. d, etc beam cable end models has shown in TABLE 2.

We implemented the above mathematical formulation into Matlab to visualize the mode shape and their respective frequencies for different end conditions of beam are given in TABLE 1. We used number of mode equal to 3 and beam cable length in meter. This mathematical model predicts that increasing frequencies are possible as number of mode becomes larger and larger.

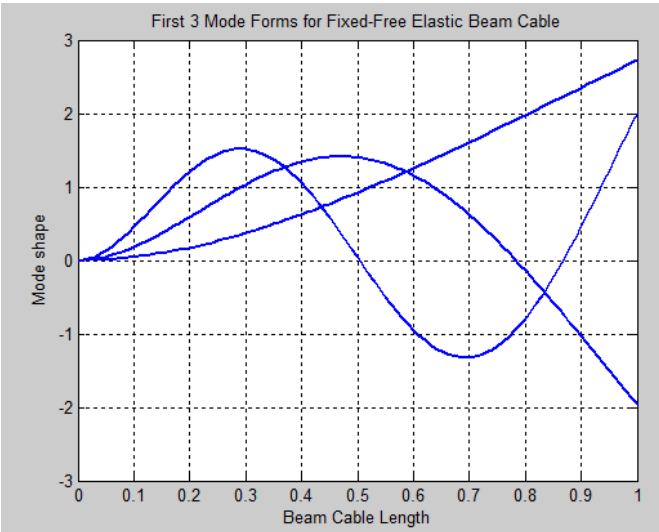

Fig. 4 The first three mode shape for fixed free elastic beam cable

To illustrate cable response due to longitudinal or parametric excitation of mainly taut mooring system, drilling riser system or tension leg tethers can be considered has shown in Table 2. This type of excitation may occur in vertical lines when the ship, buoy or platform undergoes heave motion in regular waves.

Parametric oscillator for equation (7) is solved numerically for time history. The equation can be time integrated using methods like Euler-method, Runge-Kutta method using initial boundary values. Runge-Method is the most accurate method.

From figure 7 and 8, it is clear that the amplitude of the oscillation keeps on increasing exponential, showing the instability. The plot shows that the solution moves away from the stable point. This instability is due to a phenomenon called parametric resonance. Parametric resonance takes place when the external excitation frequency approaches integral multiple of the systems natural frequency. This is a nonlinear resonance phenonmenon. In most of the cases this parametric resonance is catastrophic. This instability will not be affected by damping as well. Damping only may reduce the rate of increase of amplitude.

The solution of Mathieu equations is given figure 6.
If $(\bar{\alpha}_n, \bar{\beta}_n)$ lies in between the curve area regions, then $y_n(\tau)$ is stable, but is this parameter set is elsewhere, the responses are unstable.

We compared our result with Haines-Stability plot in reference [5] which is same as our stability diagram shown in figure 6. It is also clear the oscillator oscillates indefinitely, with large oscillator showing beating like behavior between the natural frequency of cable and external frequency. In the figure 5, 7 and 8 we have shown behavior of the oscillator without damping and this position of this point in the stability diagram in figure 6. It must be in the unstable region.

TABLE 1
CABLE BEAM FREQUENCIES (RAD/SEC)

```
Number of Requested Modes = 3

Simply Supported Frequencies

3.141593
6.283185
9.424778

Free-Free Frequencies

 4.730041
 7.853205
 10.995608

Fixed-Fixed Frequencies

 4.730041
 7.853205
 10.995608

Fixed-Free Frequencies

 1.875104
 4.694091
 7.854757
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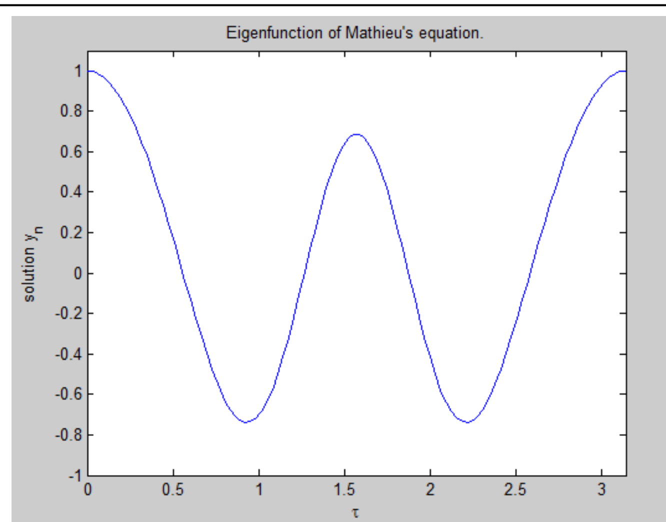

Fig. 5 stable or bounded response of cable

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TABLE2
BEAM CABLE END CONDITIONS

Fig. a Both ends simply supported beam cable model

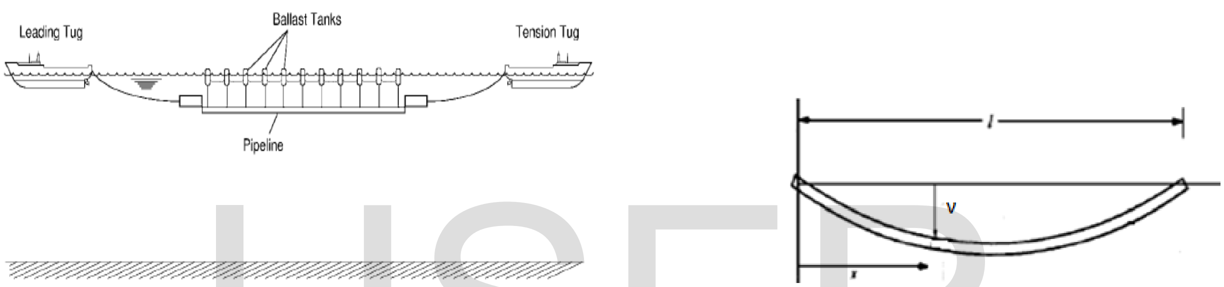

Fig. b.Free free elastic beam cable model

Fig. c. Fixed fixed beam cable model

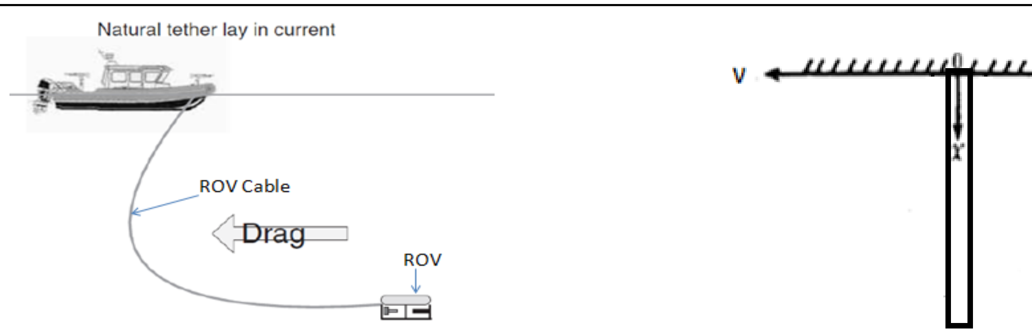

Fig. d. Fixed free beam cable model

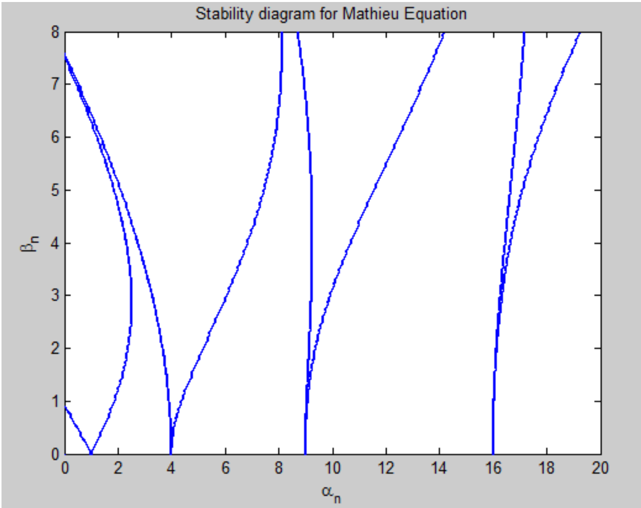

Fig.6 Stability diagram for Mathieu Equation

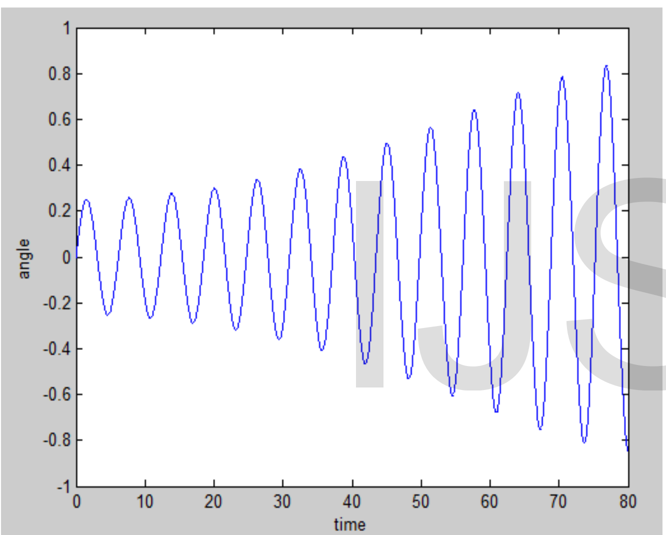

Fig. 7 Unstable or unbounded response

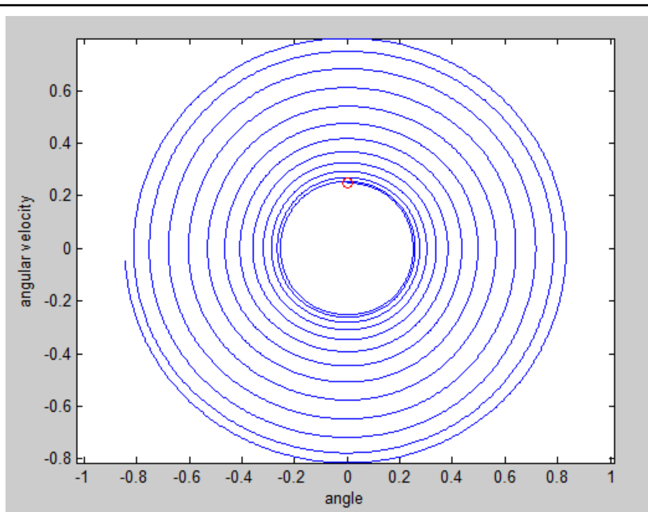

Fig. 8 Unstable or unbounded response

4 Conclusion

The purpose of the dynamic analysis of cable is to represent a reliable and safe structure in marine applications. This paper has presented cable mode shape, frequencies and response which may be a good result in reality. But we need to calculate number of modes to obtain upper limit. Also in stability diagram of cable and instability phase diagram can explain the reliable safe design parameters. We used Mathieu model for response cable response estimation and this model is able to explain nonlinear effect of parametric model but it is not exactly explained the behavior of cable in random motion as we considered as a regular heave motion. But marine cable analysis may be difficult for sever environmental condition which will predict the actual maximum limit. Therefore, the model we have presented in this paper is not able to explain the random phenomenon and random behaviors of cable which we physically expect all the time. We are also looking in our next paper to focus on stochastic analysis of dynamical cables.

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